Math 110 Sec

Midterm 3
November 30, 2004
Instructor: Charles Cuell

Nam

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All solutions are to be presented on the exam paper in the space provided. A disorganized or messy solution will result in a mark of zero for that question. Time for the exam is **80 minutes**.

(1) Compute the following. 1 mark each.

(a) $\cos(-\frac{3\pi}{4})$

$= -\frac{1}{\sqrt{2}}$

(b) $\sin(\frac{7\pi}{3}) = \frac{\sqrt{3}}{2}$

(c) $\cot(\frac{11\pi}{6}) = \frac{1}{\sqrt{3}}$

(d) $\csc(\frac{3\pi}{2})$

$= 0$

(2) Find the solution sets for the following. 1 mark each.

(a) $x^2 - 2 > 7$

$\{x \in (-\infty, -3) \cup (3, \infty)\}$

$x^2 - 2 - 7 > 0$
$x^2 - 9 > 0$
$(x+3)(x-3) > 0$

(b) $\sin 2x + \sin x = 0$, on $[0, 2\pi]$.

(3) Compute the following limits. If the limit does not exist, explain why. 1 mark each.

(a) $\lim_{x \to 1^+} \log_5(x - 1)$

$\log_{10} 100 = 2$

$10^2 = 100$

aside

Does not exist because

$\log_5 0$ doesn't exist

(b) $\lim_{x \to 0} \dfrac{1}{2 - e^x}$ $= \dfrac{1}{2 - e^0} = \dfrac{1}{2-1} = 1$

(4) Compute the derivatives of the following functions. 1 mark each.

(a) $f(x) = \pi x^4 + x^e + 1$

$f'(x) = 4\pi x^3 + ex^{(e-1)} + 0 = 4\pi x^3 + ex^{(e-1)}$

(b) $f(x) = 3^x + \log_3 x$

$f'(x) = \ln 3 + \dfrac{1}{x \ln 3}$

(c) $f(x) = \cot x$ $f'(x) = -\csc^2 x$

(d) $f(x) = (x^2 + 1)^{101}$

$f'(x) = 101(x^2+1)^{100}\, 2x$

$= 202x(x^2+1)^{100}$

(5) Find the derivative of $r(\theta) = \cos(\sec(\sin\theta))$. 2 marks.

$r'(\theta) = -\cos(\sec(\cos\theta)\tan(\cos\theta)\cdot\sin\theta)(\sec(\sin\theta))$

X

(6) Find the second derivative of $f(x) = e^{2x}\cos x$. 2 marks.

$f'(x) = e^{\ln 2x}(-\cos x)$

X

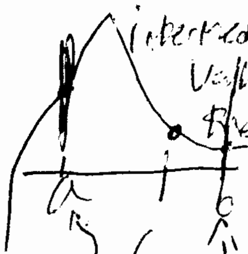
−4

(7) Find the absolute maximum and minimum of $f(x) = x^2 + x$ in the interval $[-1, 1]$. First, justify the fact that such points exist by using the appropriate theorem. *That is, name the theorem and show that it applies to this function.*

?

Not the f'n

intermediate Value theorem



$x(x+1)$ — ✓

$f'(x) = 2x + 1$

$x = \frac{-1}{2}$

x : $- - - - 0 + + + +$

$x+1$: $- - 0 + + + + + + +$

$-1 \quad 0$

$+ \; 0 \; - \; 0 \; +$

$f(-1) = (-1)^2 + (-1) = 0$

$f(0) = 0 + 0 = 0$

$f(1) = 1 + 1 = 2$

$f(\frac{-1}{2}) = \frac{1}{4} + \frac{-2}{4} = \frac{-1}{4}$

Abs min : $(\frac{-1}{2}, \frac{-1}{4})$

Abs max : $(1, 2)$

(8) Find the dimensions of a rectangle of area 100cm^2 with the smallest possible perimeter.

$A = L \cdot W \quad = \quad 100 = L \cdot W$

$A = 10 \cdot 10 = 100cm^2 \quad P = 2L + 2W \qquad W = \frac{100}{L} \qquad = W = \frac{100}{10}$

$W = 10$

$P = 2(10) + 2(10) = 40cm$

$P = 2L + 2\left(\frac{100}{L}\right) \quad \cancel{P' = 4L - a} \quad -2$

$= 2L + \frac{200}{L}$

$= 2L^2 + 200 \qquad 2L^2 = 200 \qquad L = \pm 10$

$\sqrt{L^2} = 100$

can't be minus.

(9) Suppose a particle moves along the curve $y = 1 + x^2$. If $\frac{dy}{dt} = 1$m/s, what is $\frac{dx}{dt}$ when $x = 1$ m.

$y = 1 + x^2$

$\frac{dy}{dt} = 2x \frac{dx}{dt}$

$1 = 2x$

$\frac{1 = 2\frac{dx}{dt}}{2}$

$\frac{dx}{dt} = \frac{1}{2}$